

One-dimensional model of aquifer

The linearized Boussinesq equation for the evolution with time t of the increase in hydraulic head h in unconfined aquifers and that for groundwater flow in confined aquifers is given by:

$$\frac{\partial h}{\partial t} = D \frac{\partial^2 h}{\partial x^2} + A(x, t)$$

With:

$$Q = -KD_t \frac{\partial h}{\partial x}$$

where A is the rate of seismically induced head change per unit width in the release zone, Q is discharge, X is horizontal position, K is the horizontal hydraulic conductivity, D is the hydraulic diffusivity, and Dt is the cross-sectional area of the aquifer.

Note: assume that discharge is derived from saturated flow and hence that Darcy flow applies.

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The aquifer extends from $x = 0$ at the catchment divide to $x = L$ at the channel.

Boundary conditions are $h(L, t) = 0$ and $\frac{\partial h(0, t)}{\partial x} = 0$

where recharge occurs over $0 < x < L'$. At the time of the earthquake ($t = 0$) discharge is Q_0 .

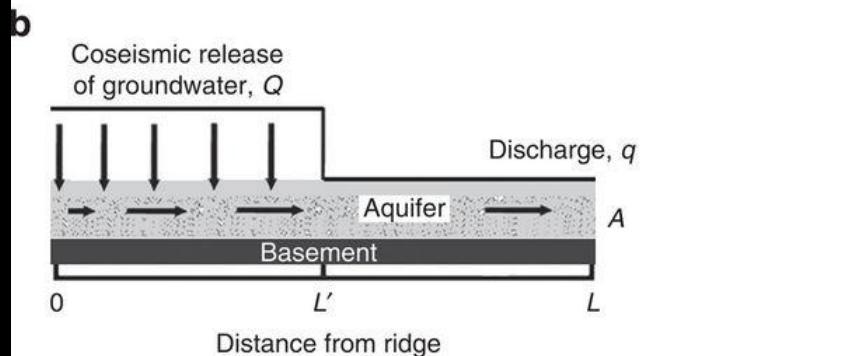
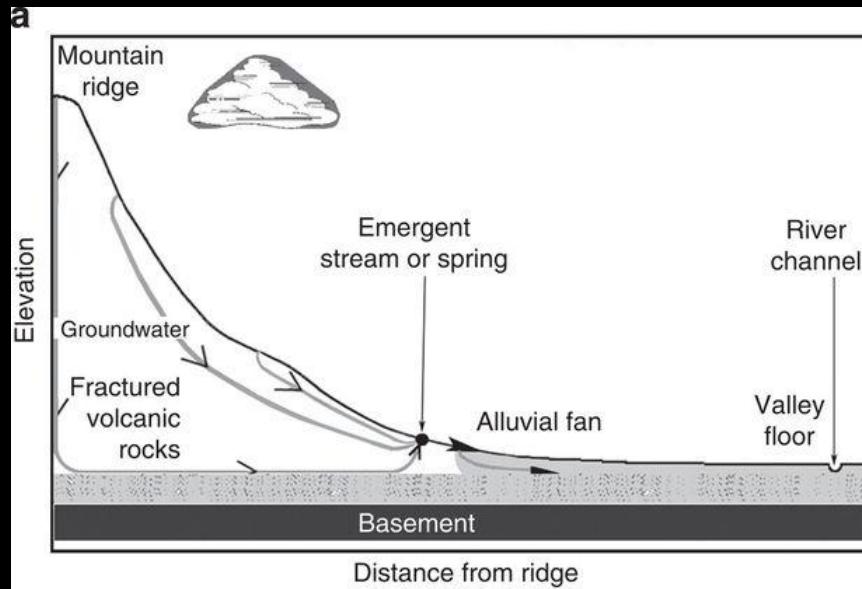
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Solution [from Wang and Manga, Nature Communications, 2015]

$$Q(t) = Q_0 e^{-\pi D t / 4L^2} + \frac{2DQ_t}{LL'} \sum_{r=1}^{\infty} (-1)^{r-1} \sin \frac{(2r-1)\pi L'}{2L} e^{-\frac{(2r-1)\pi^2 D t}{4L^2}}$$

where **Q(t)** is total water discharged by the earthquake at time t, the first term on the right-hand side describes the base flow recession in the absence of an earthquake, and the second term on the right-hand side is the excess discharge caused by the earthquake.

Fit this equation to data using **three** parameters **L'/L**, **D/L2**, and **Qt** using the nonlinear least squares Marquardt-Levenberg algorithm.



(a) Cartoon showing the conceptual model of groundwater released by earthquake from nearby mountains to recharge an underlying aquifer. Precipitation that recharges groundwater in the mountain becomes isotopically lighter at higher elevation. **(b)** Diagram showing idealized conceptual model, adapted from a, for analytical simulation. **L** and **A** are, respectively, the length and cross-sectional area of the aquifer, **L'** the length of the recharged section of the aquifer, **Q** and **q**, respectively, the coseismic release of groundwater from mountain and the discharge from aquifer. Note that **L** and **L'** may be highly variable among streams; thus the diagram is intentionally drawn at a different scale from a.

From Wang and Manga, 2015

Supplementary Note 2. Model simulation and stream discharge

We simplify the conceptual model in Fig. 4a to the configuration shown in Fig. 4b in order to apply an analytical solution⁵ (see text for discussion) to assess the excess stream discharge in response to a coseismic release of groundwater from mountains, with the latter caused by a coseismic enhanced vertical permeability.

The solution for the excess stream discharge is⁵

$$q = \frac{2DQ}{LL'} \sum_{r=1}^{\infty} (-1)^{r-1} \sin\left[\frac{(2r-1)\pi L'}{2L}\right] \exp\left[-\frac{(2r-1)^2 \pi^2 D}{4L^2} t\right]$$

where Q is the coseismic release of groundwater, L the length of the aquifer, L' the length of the recharged section of the aquifer, t the time since the earthquake, $D = K/S_s$ is the hydraulic diffusivity of the aquifer, K the hydraulic conductivity and S_s the specific storage. In the solution of the groundwater flow equation these parameters appear in three free parameters: Q , D/L^2 and

L'/L . We determine the parameters by fitting the equation to the time history of the observed excess flows using the non-linear least-squares Marquardt-Levenberg algorithm. Some of these parameters may contain large uncertainties in view of the relatively small number of data available. If uncertainties are so large that any one parameter is poorly determined, we fix L'/L to 0.1. The parameters so determined are listed together with their standard errors in Supplementary Table 5.

Assuming $L \sim 2$ km (half width of Sonoma Valley at the USGS stream gauge) and using $D/L^2 = 0.0029$ day⁻¹ for Sonoma Creek from model simulation, we obtain $D \sim 0.1$ m²/s at the basin scale.

Reference:

Wang, C. Y., & Manga, M. (2015). New streams and springs after the 2014 Mw6. 0 South Napa earthquake. *Nature communications*, 6.